Particle Accelerators Part 1

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My (main) Job: LARP

- I'm the head of the US LHC Accelerator Research Program (LARP), which coordinates US R&D related to the LHC accelerator and injector chain at Fermilab, Brookhaven, SLAC, and Berkeley (with a little at Jefferson Lab and UT Austin)
- LARP consists of
  - Accelerator Systems
    - Instrumentation
    - Beam Physics
    - Collimation
  - Magnet Systems
    - Demonstrate the viability of high gradient quadrupoles based on Nb_3Sn superconductor, rather than NbTi
  - Programmatic activities
    - Management and travel
    - Toohig Fellowship
    - Support for Long Term Visitors at CERN

NOT to be confused with this "LARP" (Live-Action Role Play), which has led to some interesting emails

Outline

- Today
  - History and movitation for accelerators
  - Basic accelerator physics concepts
- Tomorrow
  - Some “tricks of the trade”
    - Accelerator techniques
    - Instrumentation
  - Case study: The LHC
    - Motivation and choices
    - A few words about “the incident”
    - Future upgrades
  - Overview of other accelerators

Discovery: It's all about energy and collision rate

- To probe smaller scales, we must go to higher energy

$$\lambda = \frac{h}{p} \approx \frac{\left(1.2\ \mathrm{fm}\right)}{p\left(\mathrm{in\ GeV/c}\right)}$$

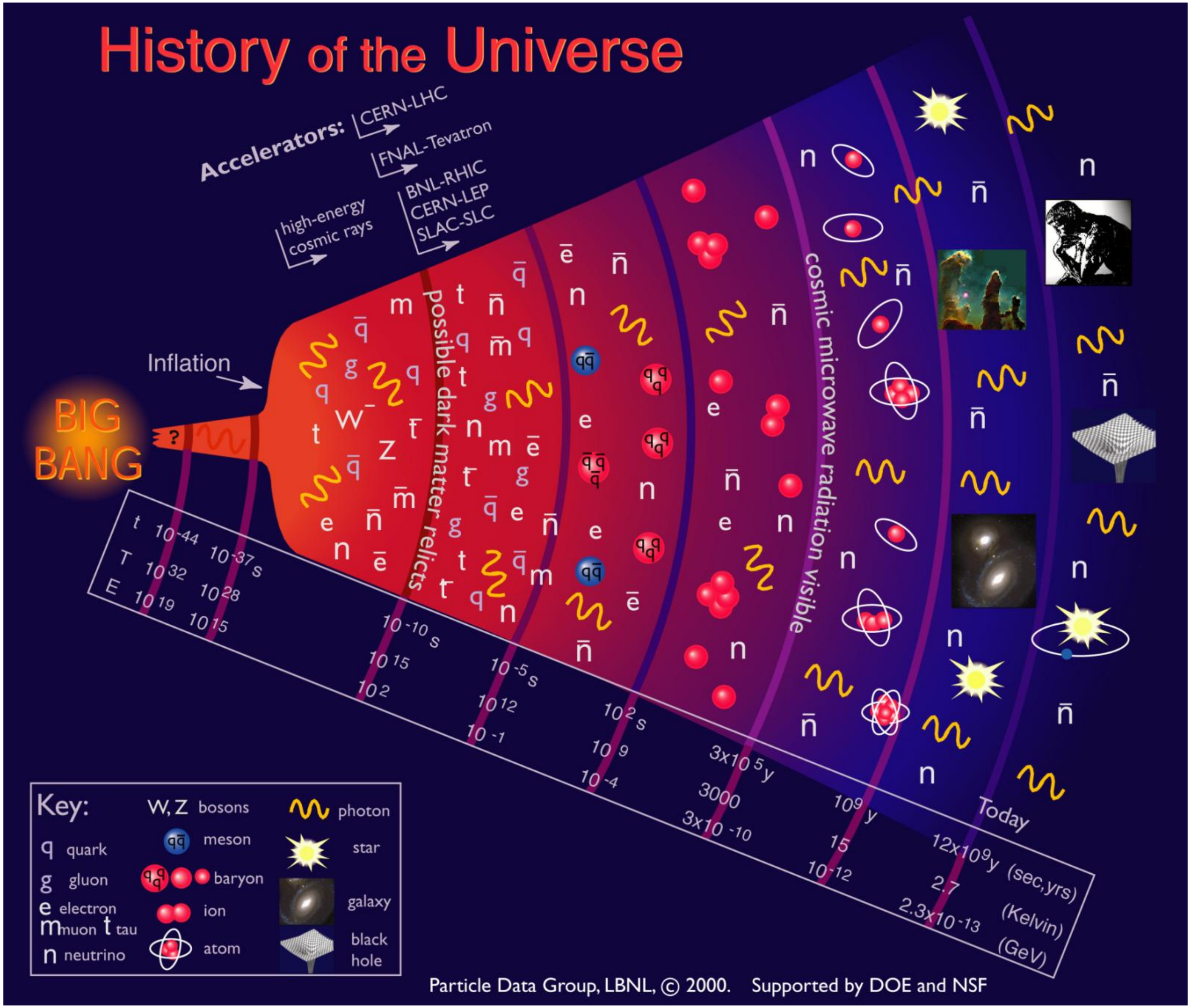
1 fm = 10^{-15} m
(Roughly the size of a proton)

- To discover new particles, we need enough energy available to create them

$$E = mc^2$$

- The rarer a process is, the more collisions (luminosity) we need to observe it.

Another way to look at it: a window back in time

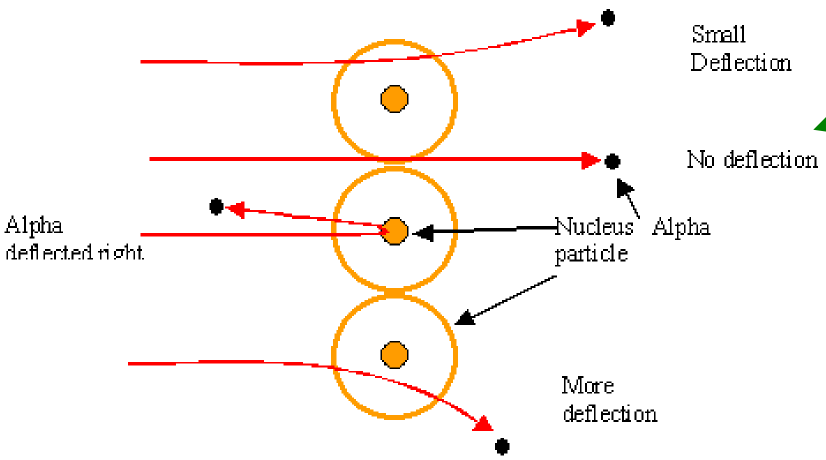


Accelerators allow us to probe down to a few picoseconds after the Big Bang!

Some pre-history

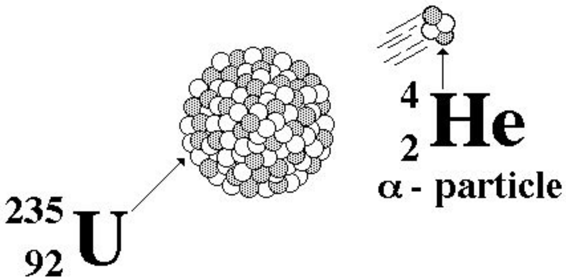
- The first artificial acceleration of particles was done using "Crookes tubes", in the latter half of the 19^{th} century
  - These were used to produce the first X-rays (1875)
  - But at the time no one understood what was going on

- The first "particle physics experiment" told Ernest Rutherford the structure of the atom (1911)



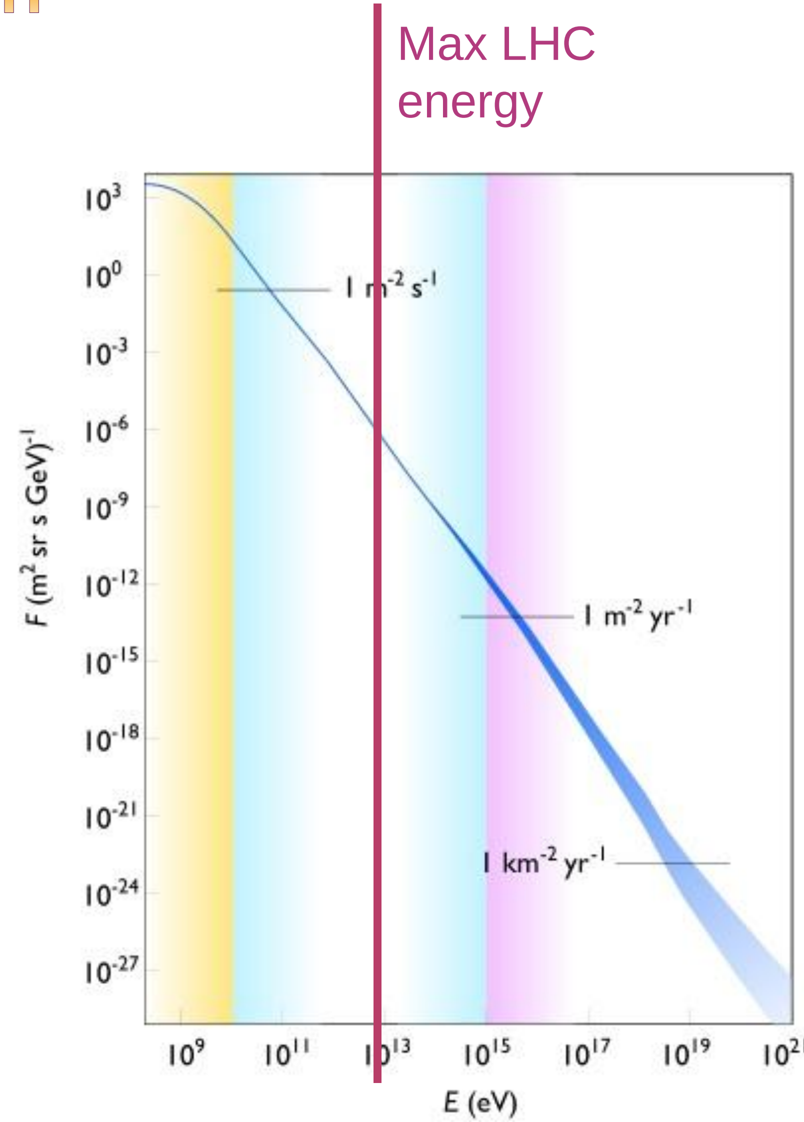
Study the way radioactive particles "scatter" off of atoms

- In this case, the "accelerator" was a naturally decaying ^{235}U nucleus



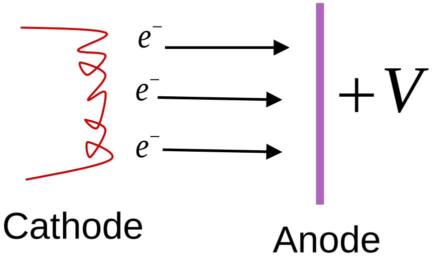
Natural particle acceleration

- Radioactive sources produce maximum energies of a few million electron volts (MeV)
- Cosmic rays reach energies of ~1,000,000,000 x LHC but the rates are too low to be useful as a study tool
  - Remember what I said about luminosity.
- On the other hand, low energy cosmic rays are extremely useful
  - But that's another talk



Man-made particle acceleration

The simplest accelerators accelerate charged particles through a *static* electric field. Example: **vacuum tubes** (or CRT TV's)



$$K = eEd = eV$$

Limited by magnitude of static field:

- TV Picture tube ~keV
- X-ray tube ~10's of keV
- Van de Graaf ~MeV's

FNAL Cockroft-Walton = 750 kV

Solutions:

- Alternate fields to keep particles in accelerating fields -> RF acceleration
- Bend particles so they see the same accelerating field over and over -> cyclotrons, synchrotrons

The Cyclotron (1930's)

- A charged particle in a uniform magnetic field will follow a circular path of radius

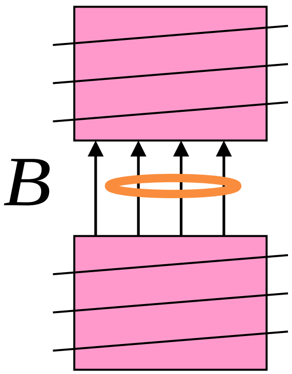
$$\rho = \frac{mv}{qB}$$

non-relativistic

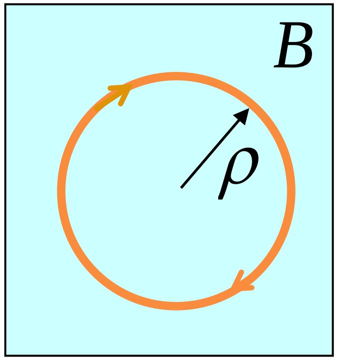
$$f = \frac{v}{2\pi\rho} = \frac{qB}{2\pi m} \text{ (constant!!)}$$

"Cyclotron Frequency"

side view

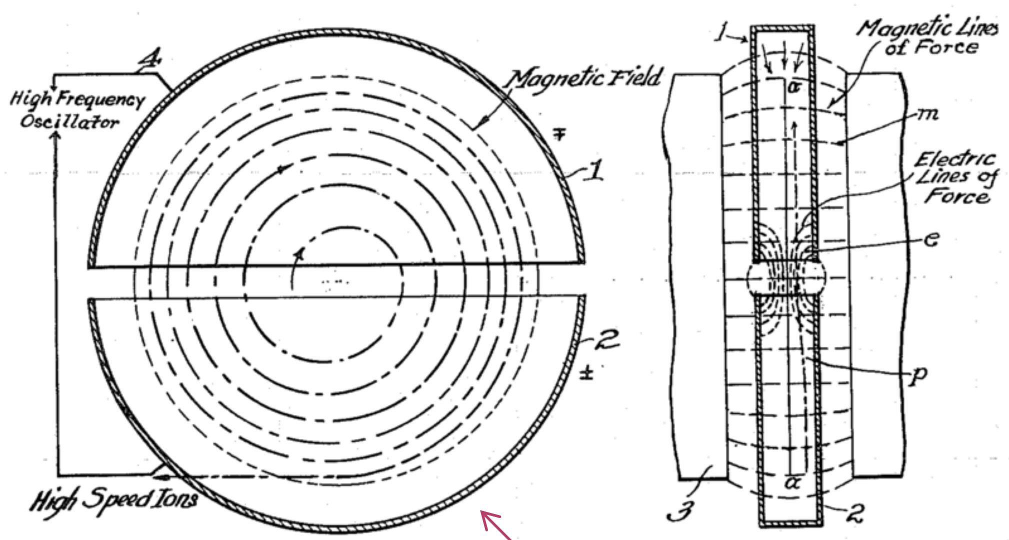


top view



For a proton:

$$f_C = 15.2 \times B[T] \text{ MHz}$$



Accelerating "DEES"

Round we go: the first cyclotrons

- ~1930 (Berkeley)
  - Lawrence and Livingston
  - K=80KeV

- 1935 - 60” Cyclotron
  - Lawrence, et al. (LBL)
  - ~19 MeV (D_2)
  - Prototype for many

Synchrocyclotron

- Cyclotrons only worked up to about 20% of the speed of light (proton energies of ~15 MeV).
- Beyond that

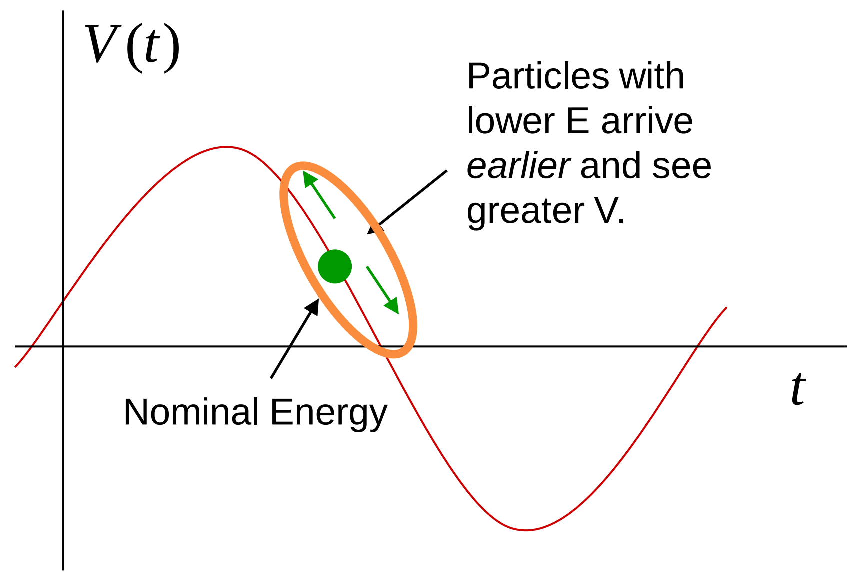
$$\rho = \frac{p}{qB} = \frac{\gamma m v}{qB}$$

$$\Rightarrow f = \frac{qB}{2\pi m\gamma} = \frac{f_C}{\gamma}$$

- As energy increases, the driving frequency must *decrease*.
- Higher energy particles take longer to go around. This has big benefits.

Phase stability!

(more about that shortly)



Synchrotrons and beam "stiffness"

- The relativistic form of Newton's Laws for a particle in a magnetic field is:

$$\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{v}\times\vec{B}\right)$$

- A particle in a uniform magnetic field will move in a circle of radius

Singly charged particles

$$\rho = \frac{p}{qB} \quad \Rightarrow \quad \boxed{\rho[\mathrm{m}] \approx \frac{p[\mathrm{MeV/c}]/300}{B[\mathrm{T}]}}$$

- In a "synchrotron", the magnetic fields are varied as the beam accelerates such that at all points $B(\vec{x},t) \propto p(t)$, and beam motion can be analyzed in a momentum independent way.
- It is usual to talk about he beam "stiffness" in T-m

$$(B\rho) = \frac{p}{q} \Rightarrow (B\rho)[\mathrm{Tm}] \approx \frac{p[\mathrm{MeV/c}]}{300}$$

Booster: (Bρ)~30 Tm
LHC : (Bρ)~23000 Tm

- Thus if at all points $B(\vec{x},t) \propto p(t)$, then the local bend radius (and therefore the trajectory) will remain constant.

Weak focusing

- Cyclotrons relied on the fact that magnetic fields between two pole faces are never perfectly uniform.
- This prevents the particles from spiraling out of the pole gap.
- In early synchrotrons, radial field profiles were optimized to take advantage of this effect, but in any weak focused beams, *the beam size grows with energy.*
- The highest energy weak focusing accelerator was the Berkeley Bevatron, which had a kinetic energy of 6 GeV
  - High enough to make antiprotons (and win a Nobel Prize)
  - It had an aperture 12"x48"!

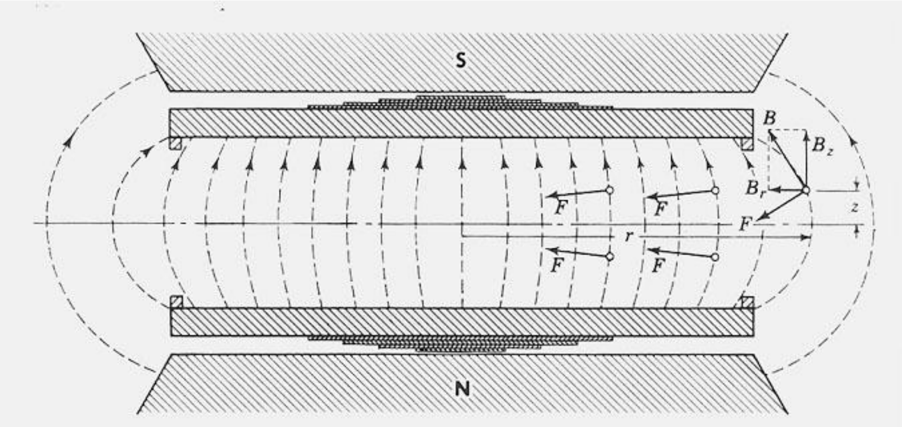
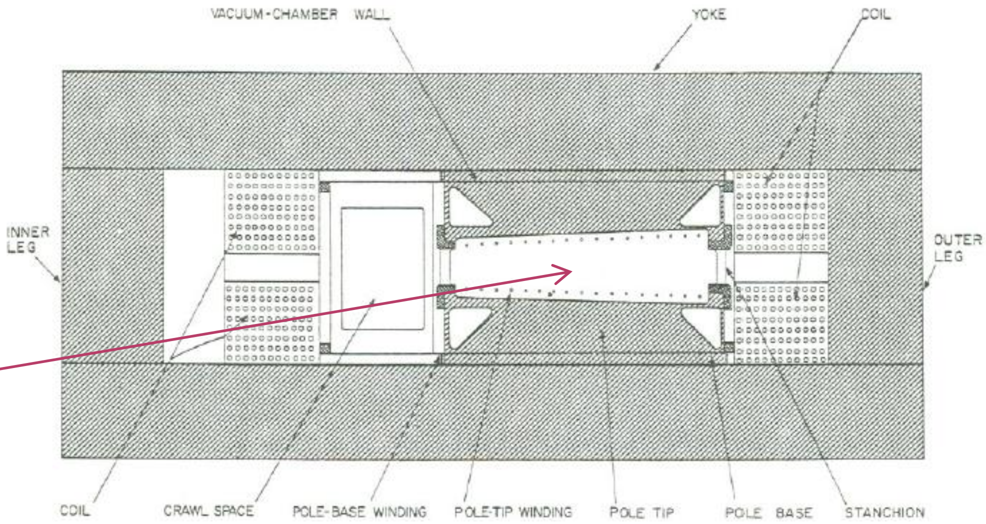


Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

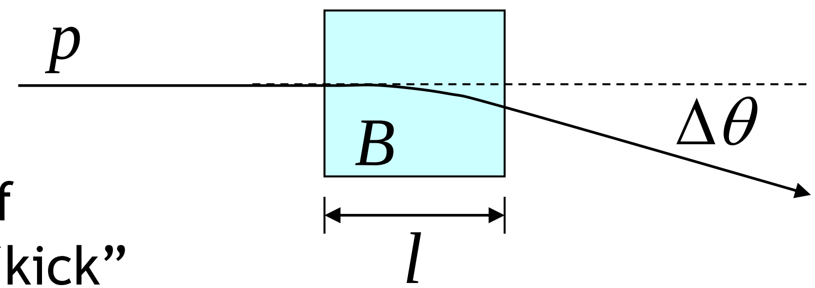


Strong focusing

- Strong focusing utilizes alternating magnetic gradients to precisely control the focusing of a beam of particles
  - The principle was first developed in 1949 by Nicholas Christophilos, a Greek-American engineer, who was working for an elevator company in Athens at the time.
  - Rather than publish the idea, he applied for a patent, and it went largely ignored.
  - The idea was independently invented in 1952 by Courant, Livingston and Snyder, who later acknowledged the priority of Christophilos’ work.
- Although the technique was originally formulated in terms of magnetic gradients, it’s much easier to understand in terms of the separate funcntions of dipole and quadrupole magnets.

Thin lens approximation and magnetic "kick"

- If the path length through a transverse magnetic field is short compared to the bend radius of the particle, then we can think of the particle receiving a transverse "kick"

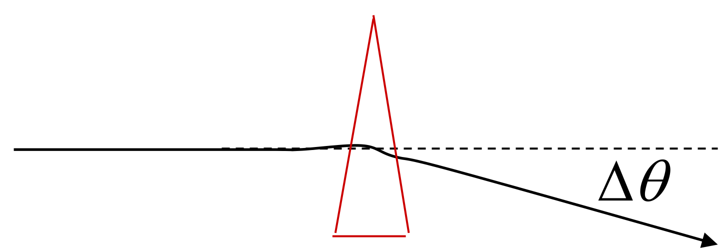


$$p_\perp \approx qvBt = qvB(l/v) = qBl$$

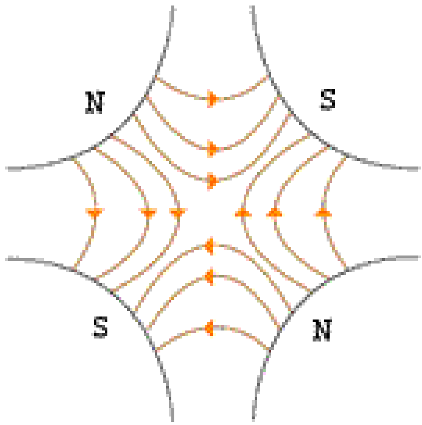
and it will be bent through small angle

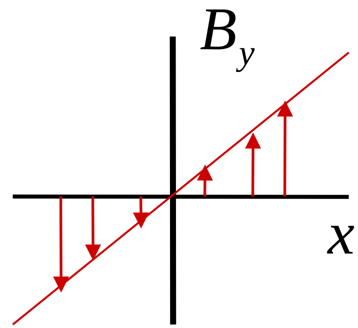
$$\Delta\theta \approx \frac{p_\perp}{p} = \frac{Bl}{(B\rho)}$$

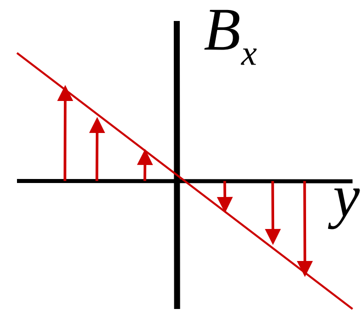
- In this "thin lens approximation", a dipole is the equivalent of a prism in classical optics.



Quadrupole magnets*

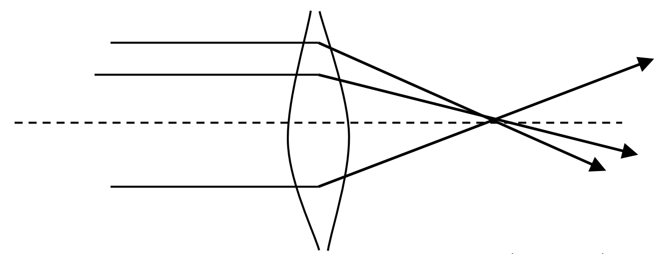






- A positive particle coming out of the page off center in the horizontal plane will experience a *restoring* kick

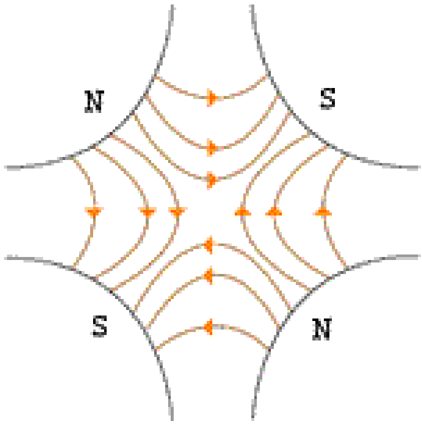
$$\Delta\theta \approx -\frac{B_x(x)l}{(B\rho)} = -\frac{B'lx}{(B\rho)}$$

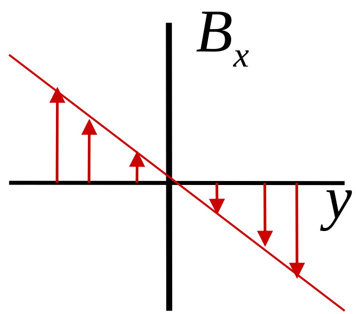


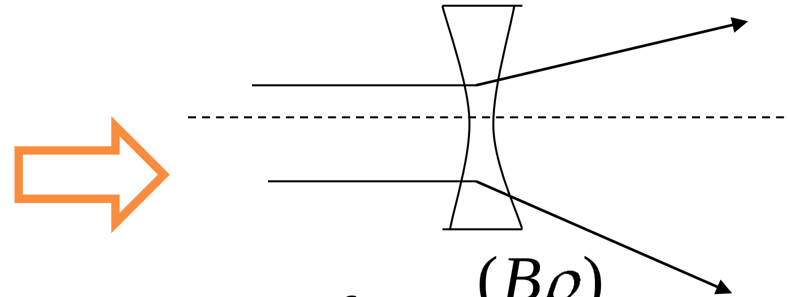
$$f = \frac{(B\rho)}{B'l}$$

*or quadrupole term in a gradient magnet

What about the other plane?



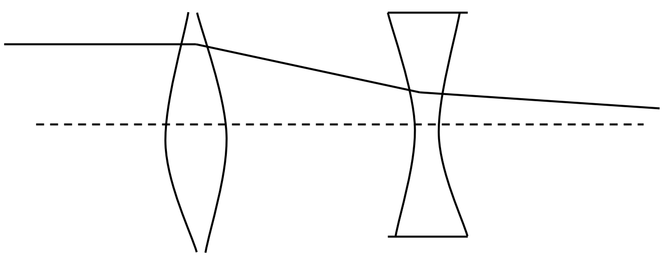


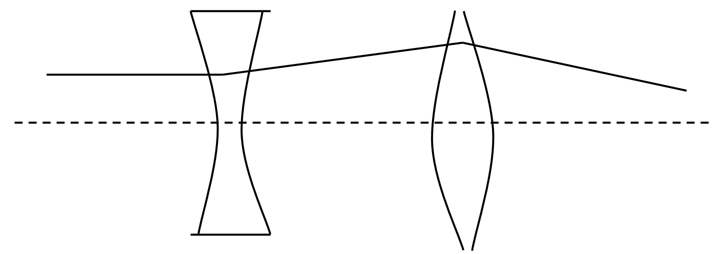


$$f = -\frac{(B\rho)}{B'l}$$

Defocusing!

Luckily, if we place equal and opposite pairs of lenses, there will be a net focusing *regardless of the order*.

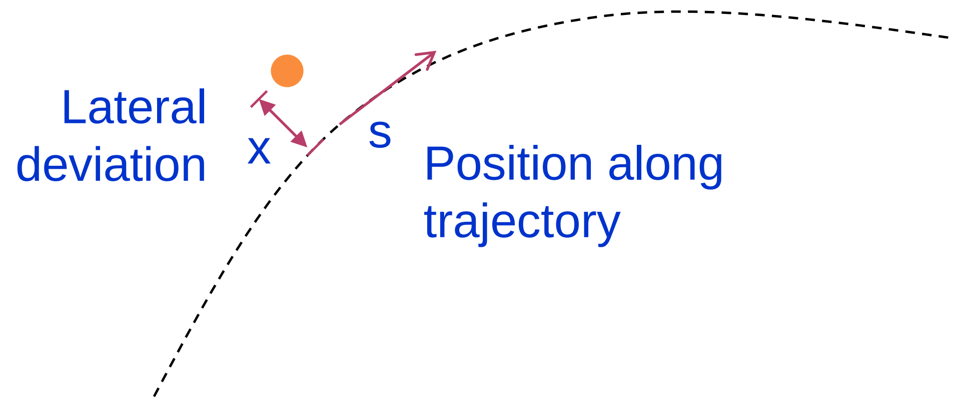


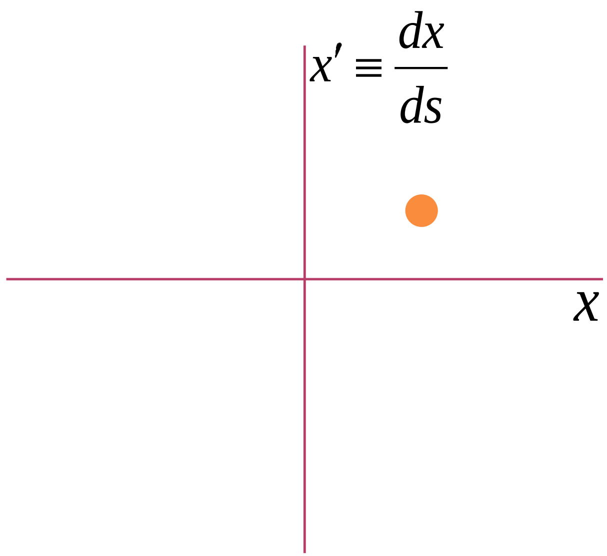


$\Rightarrow$pairs give net focusing in *both* planes -> “FODO cell”

Trajectories and phase space

- In general, we assume the dipoles define the nominal particle trajectory, and we solve for lateral deviations from that trajectory.
- At any point along the trajectory, each particle can be represented by its position in "phase space"



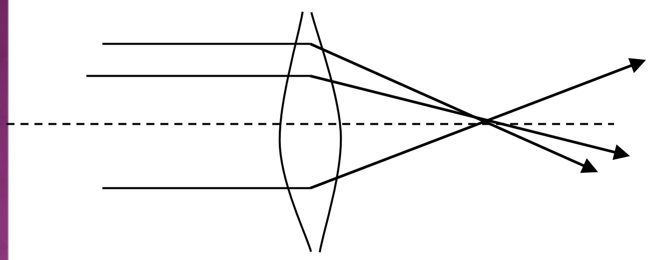


- We would like to solve for *x(s)*
- We will assume:
  - Both transverse planes are independent
    - No "coupling"
  - All particles independent from each other
    - No space charge effects

Transfer matrices

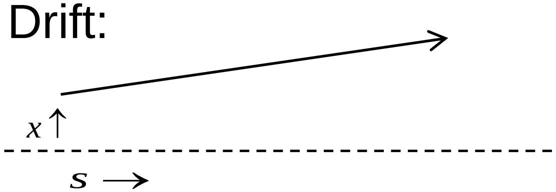
- The simplest magnetic lattice consists of quadrupoles and the spaces in between them (drifts). We can express each of these as a linear operation in phase space.

Quadrupole:



$$x = x(0)$$
$$x' = x'(0) - \frac{1}{f}x(0) \Rightarrow \begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x(0) \\ x'(0) \end{pmatrix}$$

Drift:



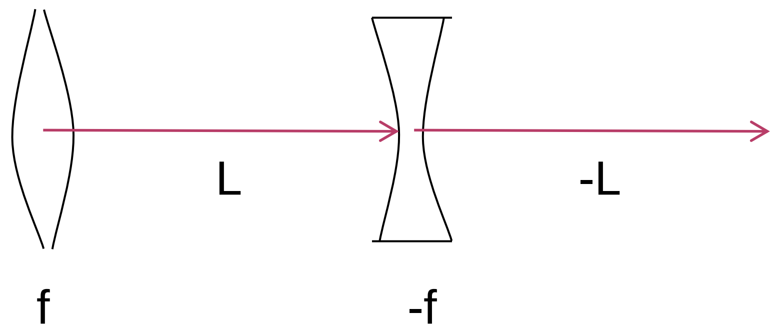
$$x(s) = x(0) + sx'(0)$$
$$x'(s) = x'(0) \Rightarrow \begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x(0) \\ x'(0) \end{pmatrix}$$

- By combining these elements, we can represent an arbitrarily complex ring or line as the product of matrices.

$$M = M_N \ldots M_2 M_1$$

Example: FODO cell

- At the heart of every beam line or ring is the "FODO" cell, consisting of a focusing and a defocusing element, separated by drifts:



- The transfer matrix is then

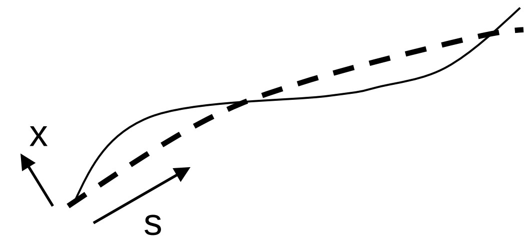
$$\Rightarrow M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ +\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L}{f} - \left(\frac{L}{f}\right)^2 & 2L + \frac{L^2}{f} \\ -\frac{L}{f^2} & 1 + \frac{L}{f} \end{pmatrix}$$

- We can build a ring out of N of these, and the overall transfer matrix will be

$$M = M_{FODO}^{N}$$

Betatron motion

- Skipping *a lot* of math, we find that we can describe particle motion in terms of initial conditions and a “beta function” β(s), which is only a function of location in the nominal path.



Lateral deviation in one plane

$$x(s) = A\left[\beta(s)\right]^{1/2}\sin\left[\psi(s)+\delta\right]$$

Phase advance

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

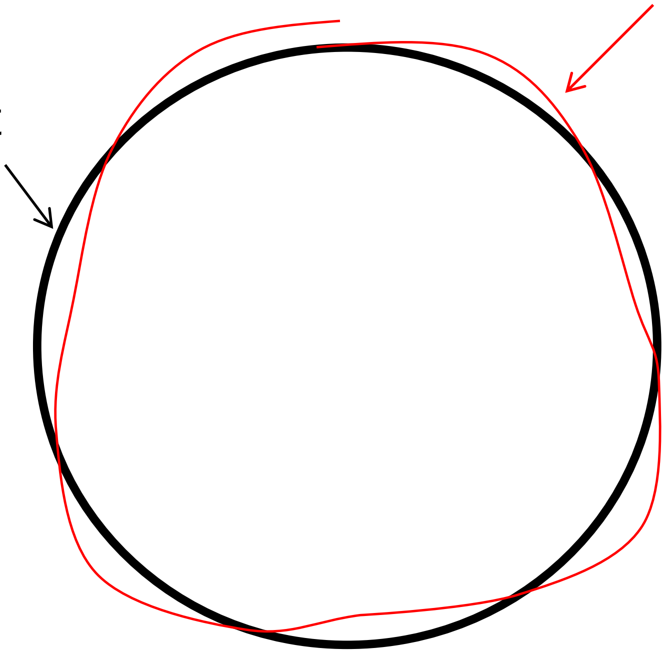
The “betatron function” $\beta(s)$ is effectively the local wavenumber and also defines the beam envelope.

Closely spaced strong quads -> small β -> small aperture, lots of wiggles

Sparsely spaced weak quads -> large β -> large aperture, few wiggles

- Minor but important note: we need constraints to define β(s)
  - For a ring, we require periodicity (of β, NOT motion): $\beta(s+C) = \beta(s)$
  - For beam line: matched to ring or source

Betatron tune



- As particles go around a ring, they will undergo a number of betatrons oscillations ν (sometimes *Q*) given by

$$\nu = \frac{1}{2\pi}\int_{s}^{s+C}\frac{ds}{\beta(s)}$$

- This is referred to as the "tune"

- We can generally think of the tune in two parts:

6.7

Integer : magnet/aperture optimization

Fraction: Beam Stability

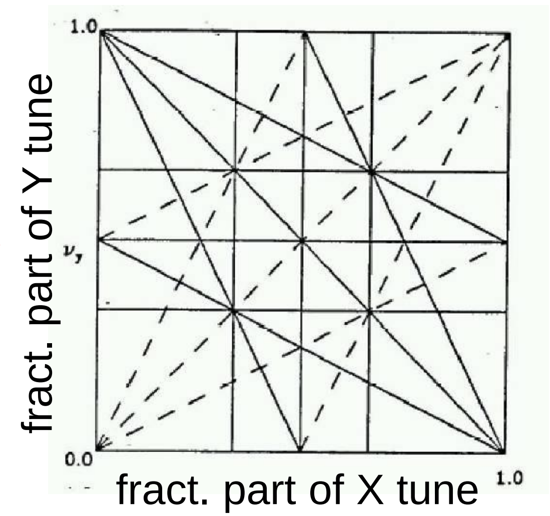
Tune, stability, and the tune plane

- If the tune is an integer, or low order rational number, then the effect of any imperfection or perturbation will tend be reinforced on subsequent orbits.
- When we add the effects of coupling between the planes, we find this is also true for *combinations* of the tunes from both planes, so in general, we want to avoid

$$k_x \nu_x \pm k_y \nu_y = \text{integer} \Rightarrow \text{(resonant instability)}$$

"small" integers

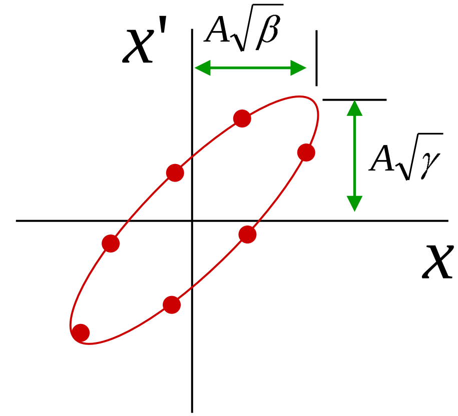
⇒Avoid lines in the "tune plane"



- Many instabilities occur when something perturbs the tune of the beam, or part of the beam, until it falls onto a resonance, thus you will often hear effects characterized by the "tune shift" they produce.

Twiss parameters: α, β, and γ

- As a particle returns to the same point *s* on subsequent revolutions, it will map out an ellipse in phase space, defined by



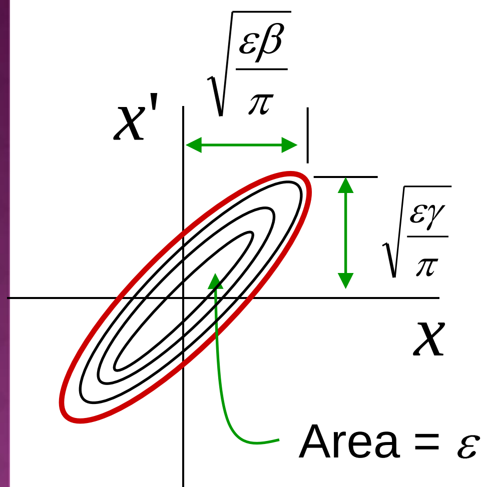
$$\gamma_T(s)x^2(s) + 2\alpha_T(s)x(s)x'(s) + \beta_T(s)x'(s)^2 = A^2$$

Twiss Parameters

$$\beta_T = \text{(betatron function)}$$
$$\alpha_T = -\frac{1}{2}\frac{d\beta_T}{ds}$$
$$\gamma_T = \frac{1+\alpha_T^2}{\beta_T}$$

- As we examine different locations on the ring, the parameters will change, but the area ($A\pi$) remains constant.

Emittance



If each particle is described by an ellipse with a particular amplitude, then an *ensemble* of particles will always remain within a bounding ellipse of a particular area:

$$\gamma_T x^2 + 2\alpha_T x x' + \beta_T x'^2 = \frac{\varepsilon}{\pi}$$

Since these distributions often have long tails, we typically define the "emittance" as an area which contains some specific fraction of the particles. Typical conventions:

Electron machines, CERN: $\varepsilon = \frac{\pi \sigma_x^2}{\beta_T}$ Contains 39% of Gaussian particles

Usually leave π as a unit, e.g. E=12 π-mm-mrad

FNAL: $\varepsilon_{95} = \frac{6\pi\sigma_x^2}{\beta_T}$ Contains 95% of Gaussian particles

Normalized emittance

As the beam accelerates, “adiabatic damping” will reduce the emittance as:

$$\varepsilon = \varepsilon_0 \frac{\gamma_0 \beta_0}{\gamma \beta}$$

The usual relativistic γ and β

$$\beta \equiv \frac{v}{c}; \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

So the “normalized emittance” will be constant: $\boxed{\varepsilon_N = \gamma\beta\varepsilon_0}$

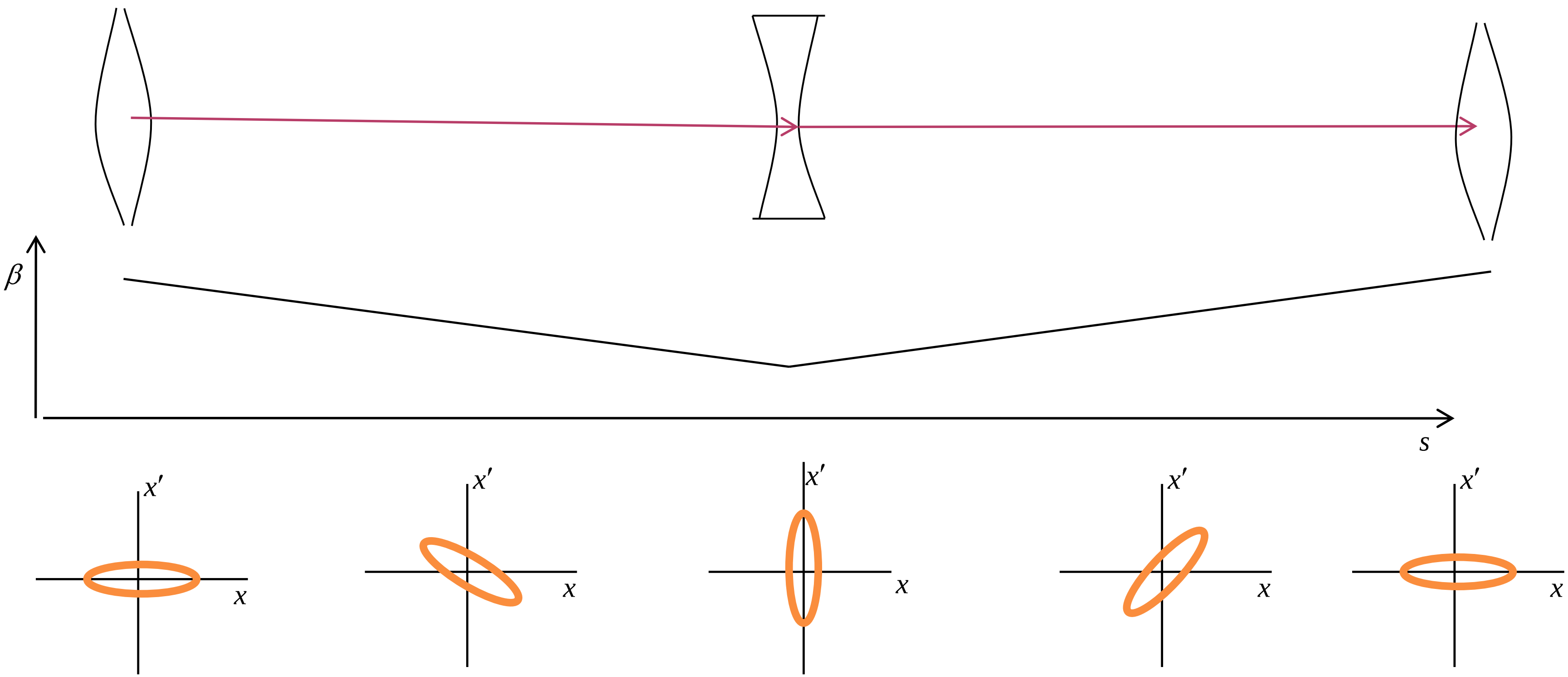
We can calculate the size of the beam at any time and with: $x(s) = \sqrt{\frac{\varepsilon \beta_T(s)}{\pi\beta\gamma}}$

Example: Fermilab Booster ⇨

			beam size [mm] (95%)	
Plane	ε_N [π-mm-mrad]	β_T [m]	Injection	Extraction
Horz	12	33.7	19.9	6.5
Horz	12	6.1	8.5	2.8
Vert	12	20.5	15.5	5.1
Vert	12	5.3	7.9	2.6

Interpreting the twiss parameters

- As particles go through the lattice, the Twiss parameters will vary periodically:



β = max α = 0 $\Rightarrow$maximum	β = decreasing α >0 $\Rightarrow$focusing	β = min α = 0 $\Rightarrow$minimum	β = increasing α < 0 $\Rightarrow$defocusing

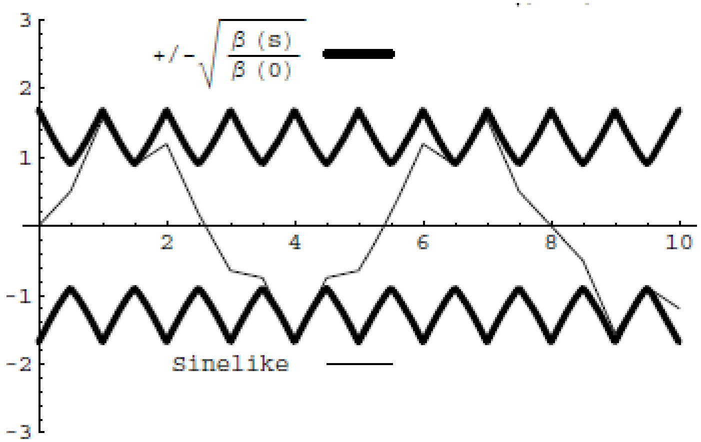
Conceptual understanding of β and ε

- In this representation, we have separated the properties of the accelerator itself (Twiss Parameters) from the properties of the ensemble (emittance). At any point, we can calculate the size of the beam by

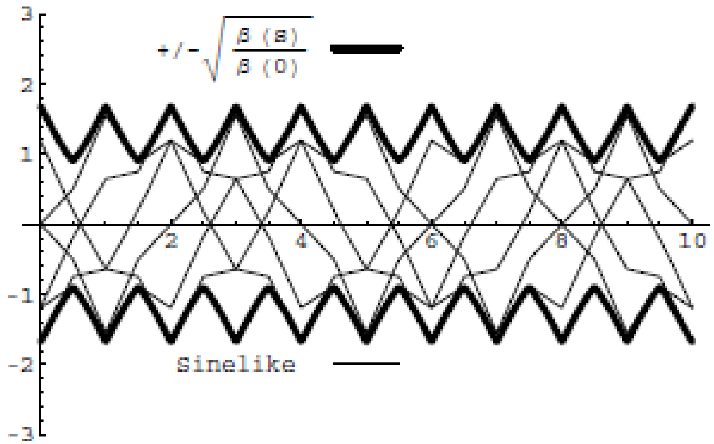
$$\sigma = \sqrt{\frac{\varepsilon \beta_T}{\pi \beta \gamma}}$$

- It's important to remember that the betatron function represents a bounding envelope to the beam motion, not the beam motion itself

Normalized particle trajectory



Trajectories over multiple turns

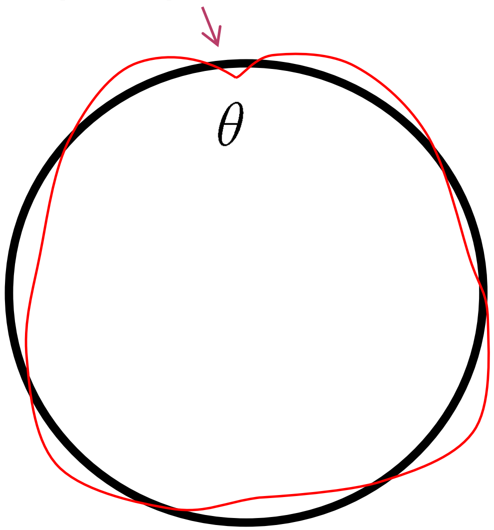


Steering errors (or corrections)

- A dipole magnet will perturb the trajectory of a beam as

$$\Delta x(s) = \theta\sqrt{\beta(s)\beta_0}\,\sin\psi(s)$$

- A dipole perturbation in a ring will cause a "closed orbit distortion" given by

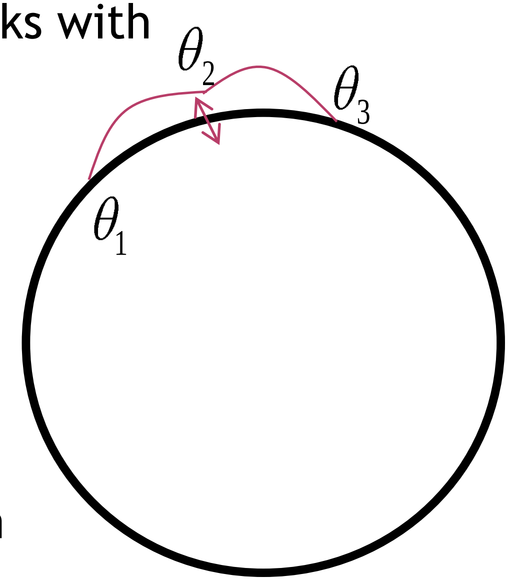


$$\Delta x(s) = \theta\frac{\sqrt{\beta(s)\beta_0}}{2\sin\pi\nu}\cos\left[|\psi(s)| - \pi\nu\right]$$

We can create a localized distortion by introducing three angular kicks with ratios

$$\theta_2 = -\theta_1\left(\frac{\beta_1}{\beta_2}\right)^{1/2}\frac{\sin\psi_{13}}{\sin\psi_{23}}$$

$$\theta_3 = \theta_1\left(\frac{\beta_1}{\beta_3}\right)^{1/2}\frac{\sin\psi_{12}}{\sin\psi_{23}}$$



- These "three bumps" are a very powerful tool for beam control and tuning

Quadrupole errors

- A single quadrupole of focal length *f* will introduce a tune shift given by

$$\Delta\nu = \frac{1}{4\pi}\frac{\beta}{f}$$

Studying these tune shifts turn out to be one very good way to measure $\beta(s)$ at quadrupole locations (more about that tomorrow).

- In addition, a small quadrupole purturbation will cause a "beta wave" distortion of the betatron function around the ring given by

$$\frac{\Delta\beta(s)}{\beta(s)} = -\frac{\beta_0}{f}\frac{1}{2\sin 2\pi\nu}\cos\left(2|\psi(s)| - 2\pi\nu\right)$$

Dispersion and chromaticity

- Up until now, we have assumed that momentum is constant.
- Real beams will have a distribution of momenta.
- The two most important parameters describing the behavior of off-momentum particles are
  - "Dispersion": describes the position dependence on momentum

$$D_x \equiv \frac{\Delta x}{(\Delta p / p)}$$

    - Most important in the bend plane
  - Chromaticity: describes the tune dependence on momentum.

$$\xi_x \equiv \frac{\Delta \nu}{(\Delta p / p)} \quad \text{OR} \quad \xi_x \equiv \frac{\Delta \nu / \nu}{(\Delta p / p)}$$

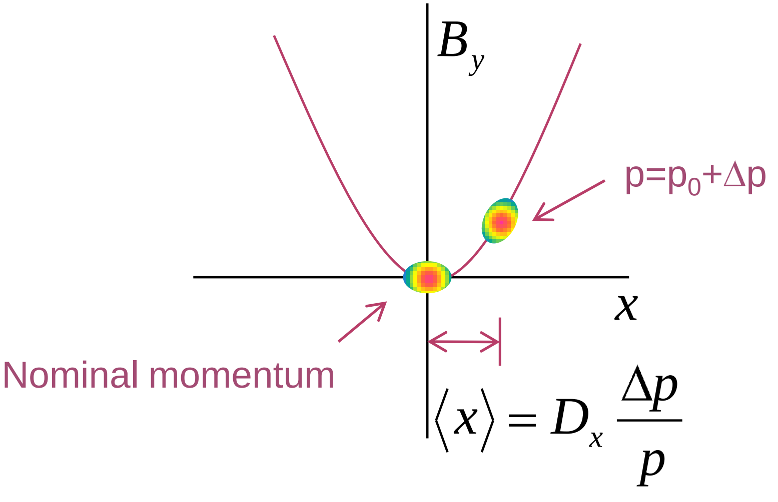
    - Often expressed in "units" of 10^{-4}

Sextupoles

- Sextupole magnets have a field (on the principle axis) given by

$$B_y(x) = B''x^2$$

- If the magnet is put in a sufficiently dispersive region, the momentum-dependent motion will be large compared to the betatron motion,



- The important effect will then be slope, which is effectively like adding a quadrupole of strength

$$B'_{eff} = \frac{1}{2}B''x = \frac{1}{2}B''D_x\frac{\Delta p}{p}$$

- The resulting tune shift will be

$$\Delta\nu = \frac{1}{4\pi}\frac{\beta}{f} = \frac{1}{8\pi}\frac{B''D_x l}{(B\rho)}\frac{\Delta p}{p}$$

$$\Rightarrow \xi = \frac{1}{8\pi}\frac{B''D_x l}{(B\rho)}$$

chromaticity

Transition

- We showed earlier that in a synchro-cyclotron, high momentum particles take longer to go around.
  - This led to the initial understanding of phase stability during acceleration.
- In a synchrotron, two effects compete

Path length

Velocity

$$\tau = \frac{L}{v}$$

"momentum compaction factor": a constant of the lattice. *Usually* positive

Momentum dependent "slip factor"

- This means that at the slip factor will change sign for

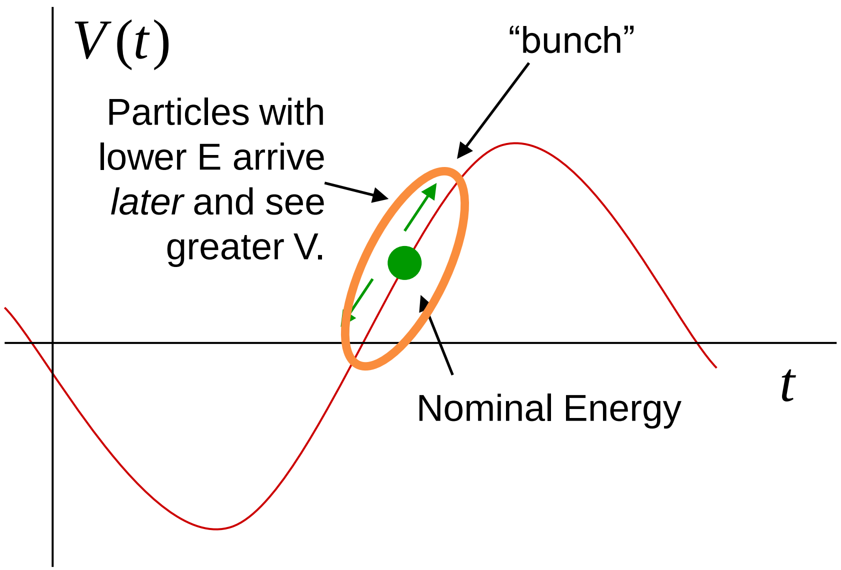
$$\gamma > \frac{1}{\sqrt{\alpha}} \equiv \gamma_t$$

"transition" gamma

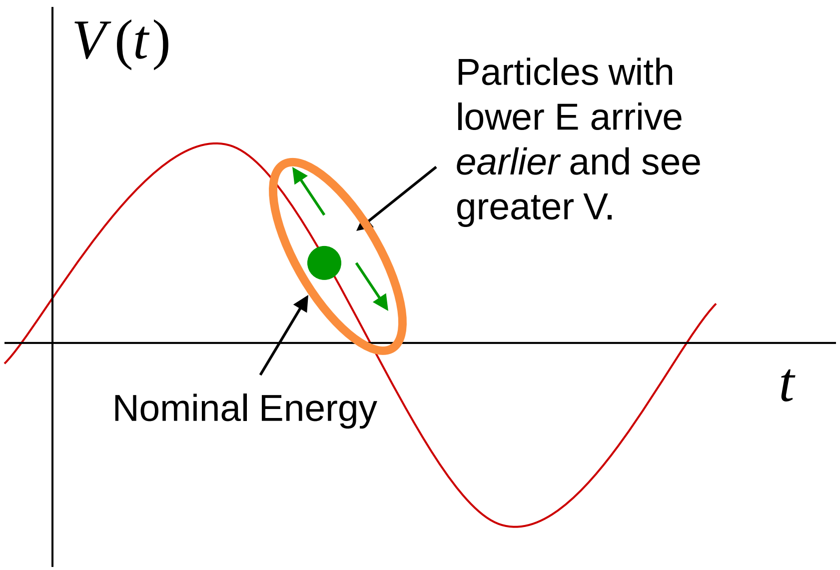
Transition and phase stability

- The sign of the slip factor determines the stable region on the RF curve.

Below γ_t: velocity dominates



Above γ_t : path length dominates



- Somwhat complicated phase manpulation at transition, which can result in losses, emittance growth, and instability
- For a simple FODO ring, we can show that

$$\gamma_t \approx \nu$$

  - Never a factor for electrons!
- Rings have been designed (but never built) with $\alpha<0 \Rightarrow \gamma_t$ imaginary

Digression: Floquet transformation

- Recall that particles in an accelerator undergo "pseudo-harmonic" motion

$$x(s) = A\left(\beta(s)\right)^{1/2}\sin\left(\psi(s)+\delta\right)$$

- Introducing the following transformation

$$\eta \equiv \frac{x}{\sqrt{\beta}}$$

$$\phi \equiv \frac{1}{\nu}\int_0^s \frac{ds}{\beta}$$

- allows the representation a lattice as a harmonic oscillator

Ideal, linear lattice

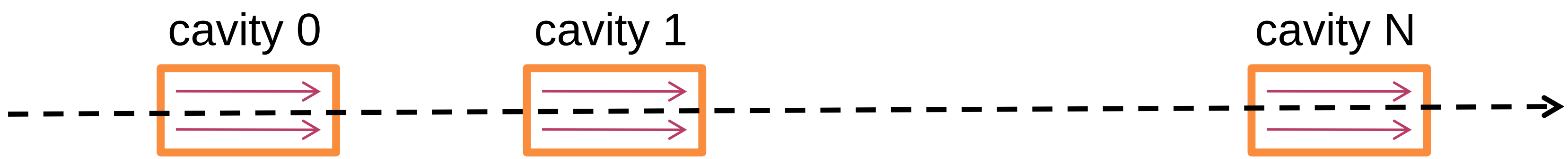
$$\frac{d^2\eta}{d\phi^2}+\nu^2\eta=\nu^2\beta^{3/2}\frac{\Delta\beta}{B\rho}$$

Driving terms

- Essentially all analytical calculations of accelerator dynamics are done in this way
  - But we won't do any

Longitudinal motion

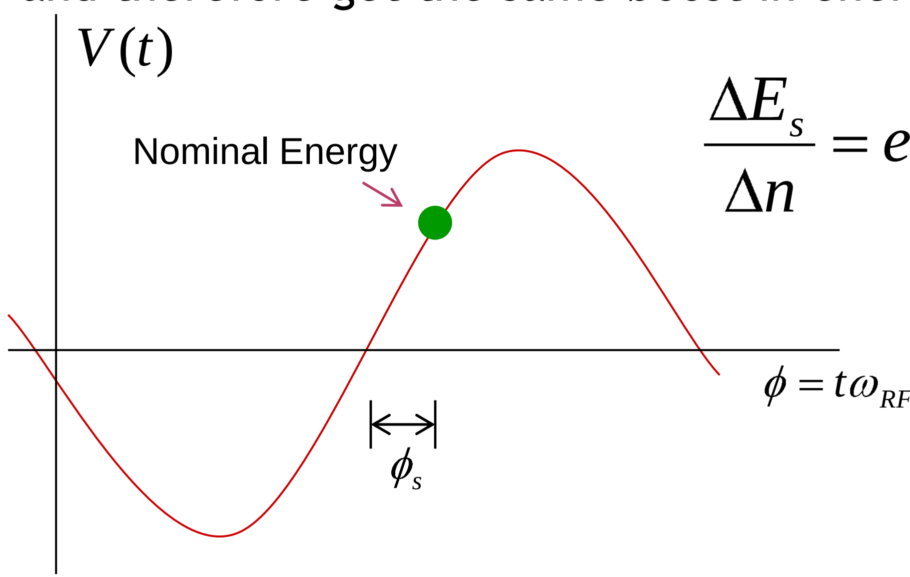
- We will generally accelerate particles using structures that generate time-varying electric fields (RF cavities), either in a linear arrangement



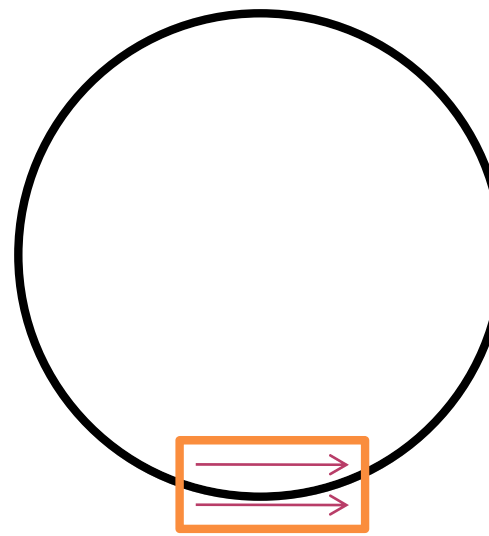
$E = E(t) = E_0 \sin \varpi t + \delta_0$ $\quad$ $E = E(t) = E_0 \sin \varpi t + \delta_1$ $\quad$ $E = E(t) = E_0 \sin \varpi t + \delta_N$

or located within a circulating ring

- In both cases, we want to phase the RF so a nominal arriving particle will see the same accelerating voltage and therefore get the same boost in energy



$$\frac{\Delta E_s}{\Delta n} = eV_0 \sin \phi_s$$

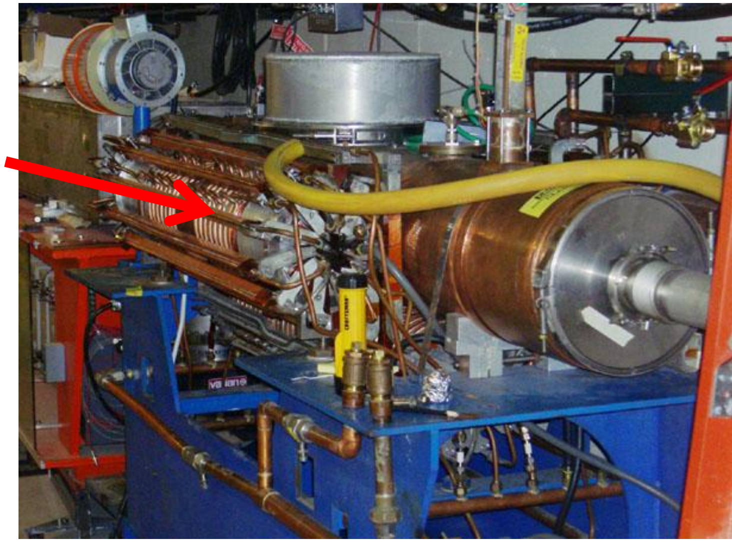


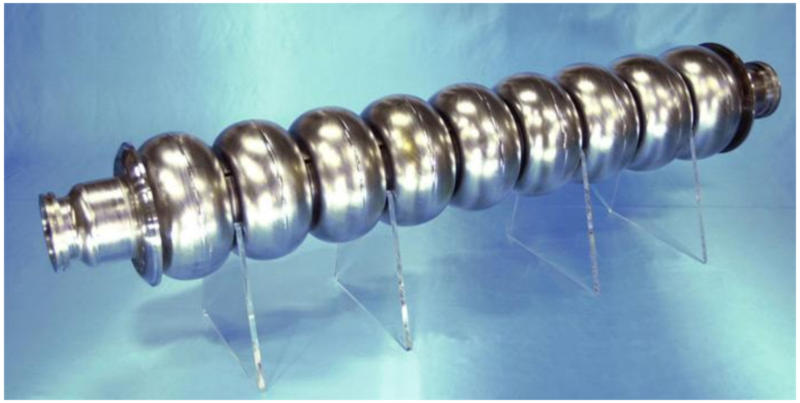
Examples of accelerating RF structures

Fermilab Drift Tube Linac (200MHz): oscillating field uniform along length

37->53MHz Fermilab Booster cavity

Biased ferrite frequency tuner

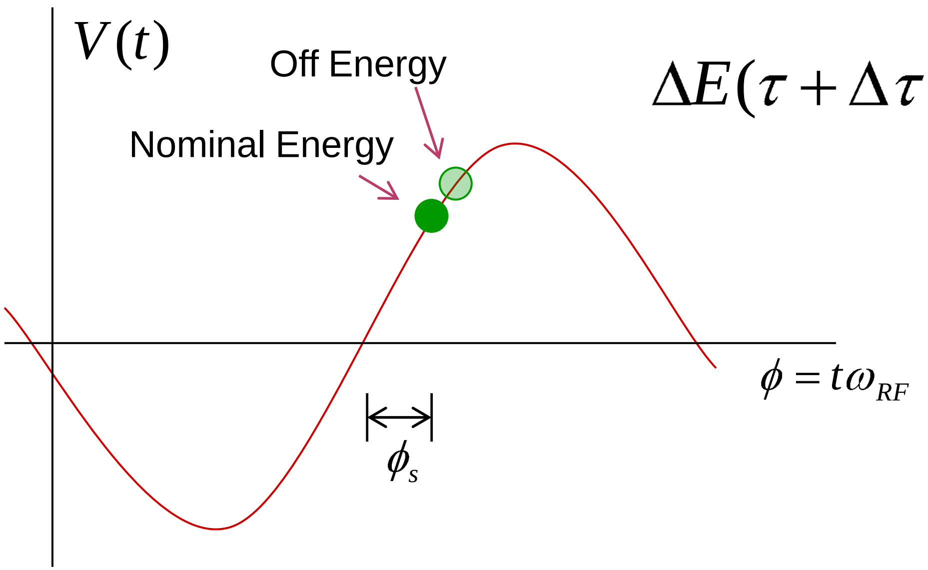




ILC prototype elipical cell “π-cavity” (1.3 GHz): field alternates with each cell

Phase stability

- A particle with a slightly different energy will arrive at a slightly different time, and experience a slightly different acceleration



$$\Delta E(\tau + \Delta\tau) \approx eV_0(\sin\phi_s + \varpi_{RF}\cos\phi_s\Delta\tau)$$
$$= \Delta E_s + \varpi_{RF} eV_0 \cos\phi_s \Delta\tau$$

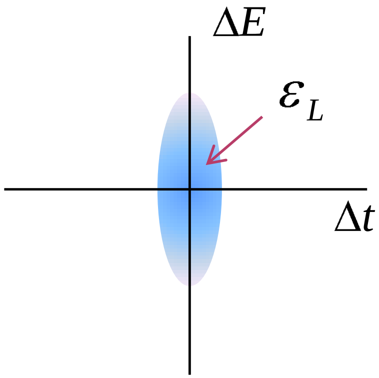
$$\frac{\Delta\tau}{\tau} = \eta\frac{\Delta p}{p} = \eta\frac{1}{\beta^2}\frac{\Delta E}{E}$$

- If $\eta\cos\phi_s < 0$ then particles will stably oscillate around this equilibrium energy with a “synchrotron frequency” and “synchrotron tune”

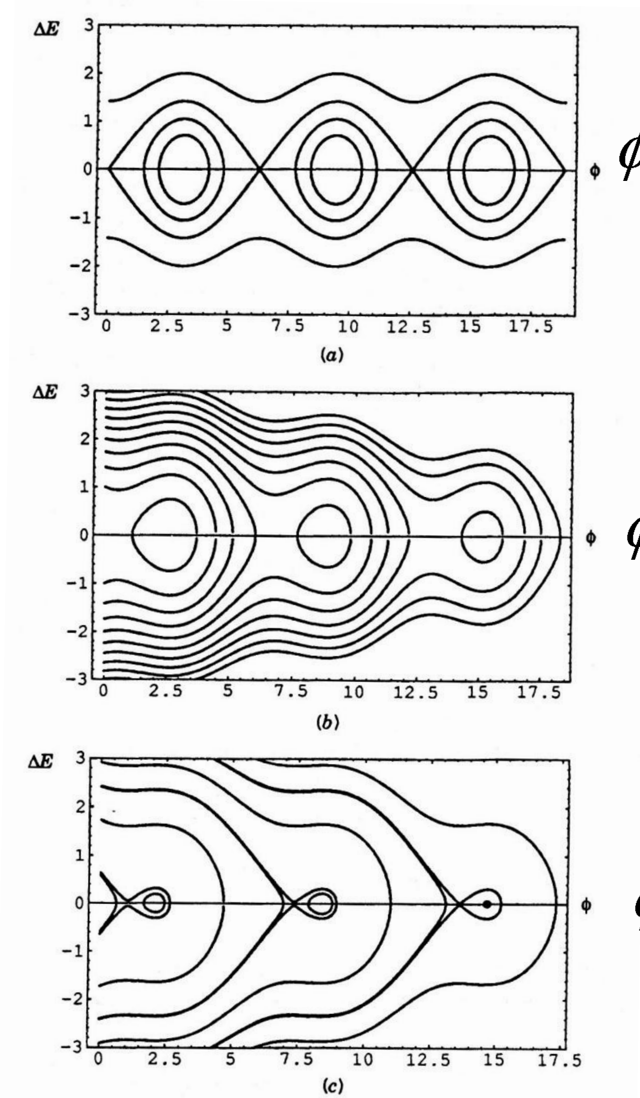
$$\Omega_s = \sqrt{-\frac{\eta\varpi_{RF} eV_0\cos\phi_s}{\tau\beta^2 E_s}};\ \nu_s = \frac{\Omega_s\tau}{2\pi} << 1$$

Accelerating phase and stability

- The accelerating voltage grows as $\sin\phi_s$, but the stable bucket area shrinks
- Just as in the transverse plane, we can define a phase space, this time in the Δt-ΔE plane



Area = "longitudinal emittance" (usually in eV-s)



$\phi_s = 0$

$\phi_s = 30°$

$\phi_s = 60°$

- As particles accelerate or accelerating voltage changes

$$\Delta E_{\max} \propto \left(V_0 \beta^2 \gamma^3\right)^{\frac{1}{4}}$$

$$\Delta t_{\max} \propto \left(V_0 \beta^2 \gamma^3\right)^{-\frac{1}{4}}$$

$$\varepsilon_L \propto \Delta E_{\max} \Delta t_{\max} = \text{constant}$$

The Case for Colliding Beams

- For a relativistic beam hitting a fixed target, the center of mass energy is:

$$E_{\mathrm{CM}} = \sqrt{2E_{\mathrm{beam}} m_{\mathrm{target}} c^2}$$

- On the other hand, for colliding beams (of equal mass and energy):

$$E_{\mathrm{CM}} = 2E_{\mathrm{beam}}$$

- To get the 14 TeV CM design energy of the LHC with a single beam on a fixed target would require that beam to have an energy of 100,000 TeV!
  - *Would require a ring 10 times the diameter of the Earth!!*

Luminosity

The relationship of the beam to the rate of observed physics processes is given by the "Luminosity"

$$R = L\sigma$$

Rate → R

"Luminosity" → L

Cross-section ("physics") → σ

Standard unit for Luminosity is $cm^{-2}s^{-1}$

For fixed (thin) target:

$$R = N\rho_n t\sigma \Rightarrow L = N\rho_n t$$

Incident rate → N

Target number density → ρ_n

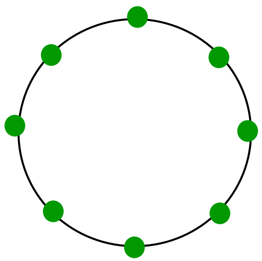
Target thickness → t

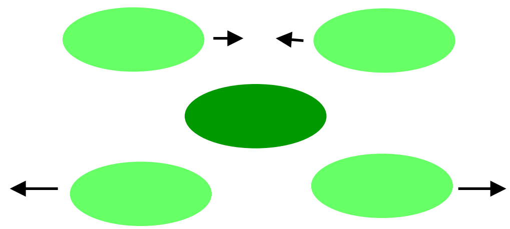
Example: MiniBooNe primary target:

$$L \approx 10^{37}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Colliding Beam Luminosity

Circulating beams typically "bunched"





(number of interactions)

$$= \left(\frac{N_1}{A}\right) N_2 \sigma$$

Cross-sectional area of beam

Total Luminosity:

$$L = \left(\frac{N_1 N_2}{A}\right) r_b = \left(\frac{N_1 N_2}{A}\right) n \frac{c}{C}$$

Number of bunches

Circumference of machine

Record e+e- Luminosity (KEK-B): 1.71E34 $cm^{-2}s^{-1}$

Record Hadronic Luminosity (Tevatron): 4.03E32 $cm^{-2}s^{-1}$

LHC Design Luminosity: 1.00E34 $cm^{-2}s^{-1}$

Luminosity: cont'd

- For equally intense Gaussian beams

Collision frequency

Particles in a bunch

$$L = f\frac{N_b^{\,2}}{4\pi\sigma^2}R$$

Geometrical factor:
- crossing angle
- hourglass effect

Transverse size (RMS)

- Expressing this in terms of our usual beam parameters

$$L = f_{rev}\frac{1}{4\pi}nN_b^2\frac{\gamma}{\beta^*\varepsilon_N}R$$

Revolution frequency

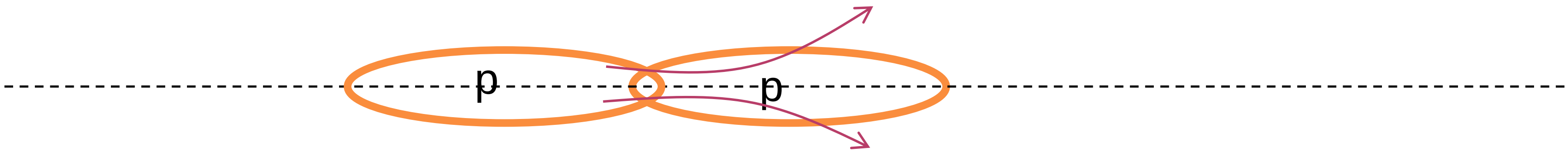
Number of bunches

Normalized emittance

Betatron function at collision point

Beam-beam Effects (NOTE correction)

- It seems like we want to get the beam as small and intense as possible, but we have to remember that the beams influence each other.



- A beam passing through another beam will see either a focusing (pBar-p) or defocusing (p-p) field, resulting in a tune spread on a scale

Number of collisions per bunch

Classical electron radius

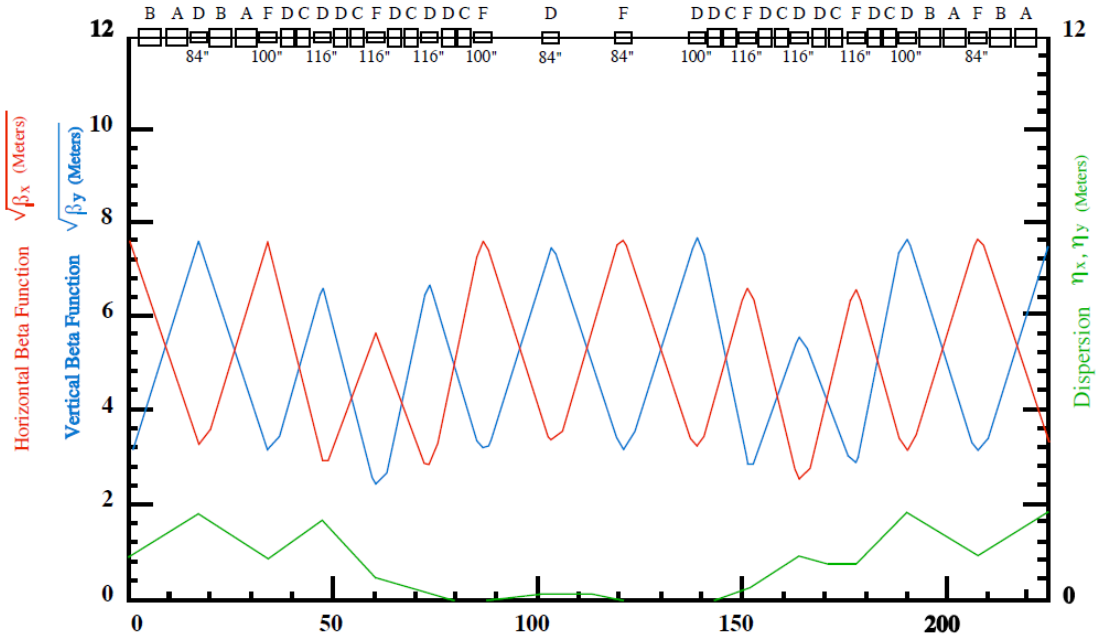
Particles in a bunch

$$\Delta\nu = \frac{r_0 n N_b}{4\varepsilon_N}$$

- Keep in mind, this is the maximum of a spread of tunes, so it they can't be simply compensated
- Typical maximum values are ~.02
- This limits the beam "brightness" (N_b/ε_N) to $\frac{N_b}{\varepsilon_N} <= \frac{4\Delta\nu_{max}}{n r_0}$

Limits to β^*

- An ordinary synchrotron lattice is characterized by FODO cells, in which vertical maxima correspond to horizontal minima, and vice versa



Lattice of the Fermilab Main Injector

- Creating a minimum in *both* planes can in general be solved by putting a triplet of quads on either side of the interaction region
  - Low beta "insertion"
  - Constrain lattice functions and phase advance to match "missing" period.

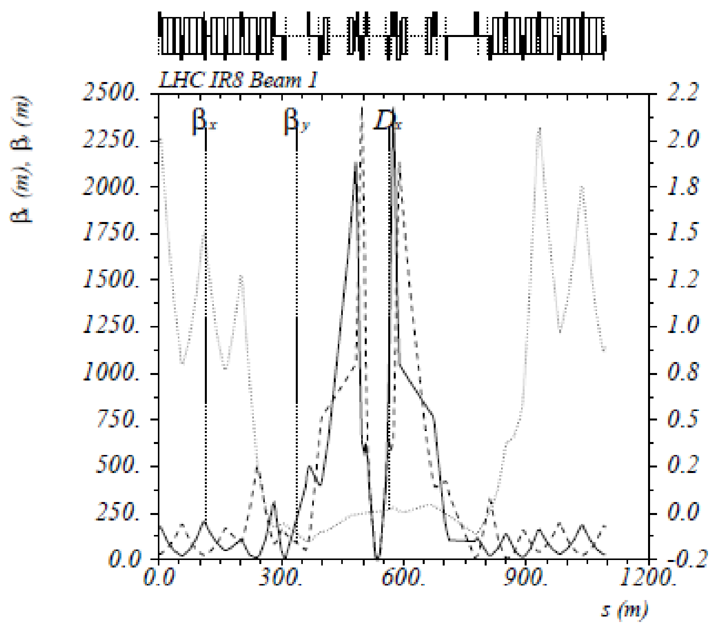
Optics near an interaction region

- Near a beam waist, the beta function will evolve quadratically

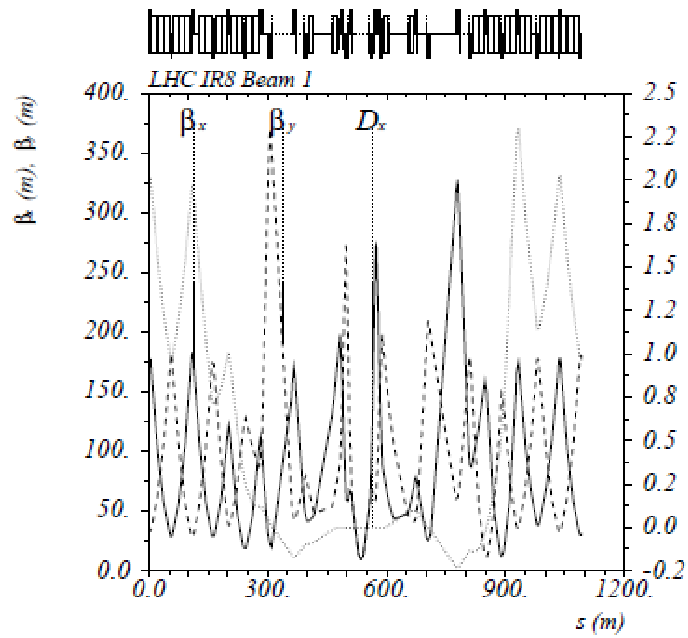
$$\beta(\Delta s) = \beta^* + \frac{1}{\beta^*}\Delta s^2$$

- Since there is a limit to how close we can put the focusing triplets, the smaller the β*, the larger the β (aperture) at the focusing triplet, and the stronger that triplet must be, which is limited by magnet technology

LHC collision region at 7 TeV region (β*=55cm)

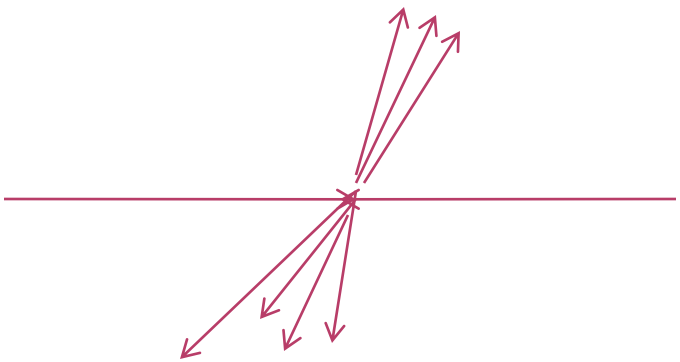


At 450 GeV (β*=10m)

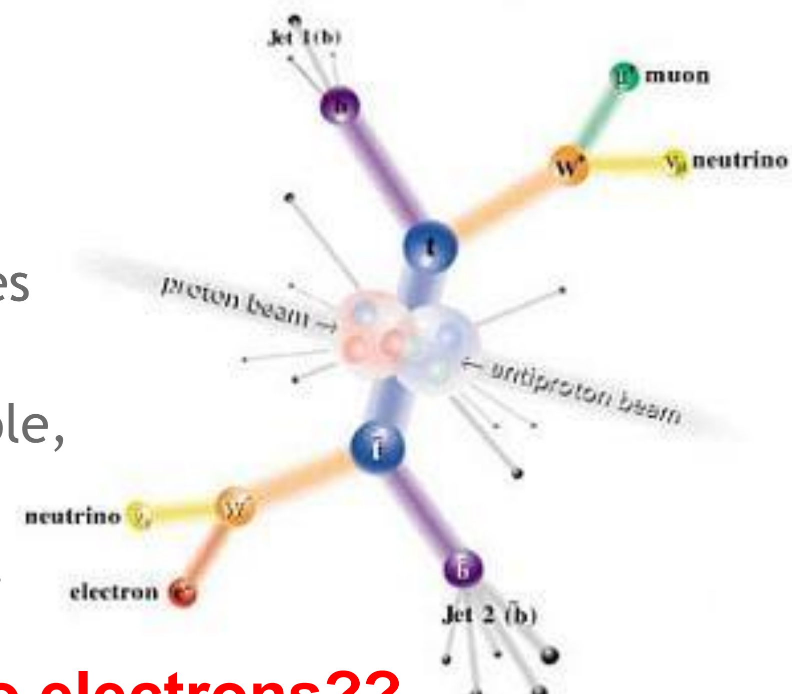


Must relax optics at injection so particles can clear triplets, then “squeeze” later.

Electrons (leptons) vs. Protons (hadrons)



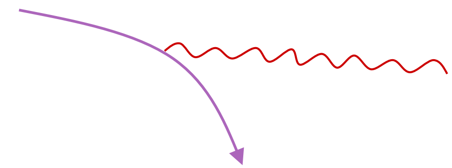
- Electrons are point-like
  - Well-defined initial state
  - Full energy available to interaction
  - Can calculate from first principles
  - Can use energy/momentum conservation to find “invisible” particles.
- Protons are made of quarks and gluons
  - Interaction take place between these consituents.
  - At high energies, virtual “sea” particles dominate
  - Only a small fraction of energy available, not well-defined.
  - Rest of particle fragments -> big mess!



So why don’t we stick to electrons??

Synchrotron Radiation: a blessing and a curse

As the trajectory of a charged particle is deflected, it emits "synchrotron radiation"



Radius of curvature

$$P = \frac{1}{6\pi\varepsilon_0}\frac{e^2 c}{\rho^2}\left(\frac{E}{m}\right)^4$$

An electron will radiate about *10^{13} times more power* than a proton of the same energy!!!!

- **Protons:** Synchrotron radiation does not affect kinematics very much
- **Electrons:** Beyond a few MeV, synchrotron radiation becomes very important, and by a few GeV, it dominates kinematics
  - Good Effects:
    - Naturally "cools" beam in all dimensions
    - Basis for light sources, FEL's, etc.
  - Bad Effects:
    - Beam pipe heating
    - Exacerbates beam-beam effects
    - **Energy loss ultimately limits circular accelerators**

Practical consequences of synchrotron radiation

- Proton accelerators
  - Synchrotron radiation not an issue to first order
  - Energy limited by the maximum feasible size and magnetic field.
- Electron accelerators
  - Recall

$$P=\frac{1}{6\pi\varepsilon_0}\frac{e^2c}{\rho^2}\left(\frac{E}{m}\right)^4\propto\left(\frac{E^2}{\rho}\right)^2$$

  - To keep power loss constant, radius must go up as the *square* of the energy (weak magnets, BIG rings):
    - The LHC tunnel was built for LEP, and e^+e^- collider which used the 27 km tunnel to contain 100 GeV beams (1/70th of the LHC energy!!)
    - Beyond LEP energy, circular synchrotrons have no advantage for e^+e^-
      - -> International Linear Collider (but that's another talk)

Review: glossary of terms

- **"RF cavity"**: resonant electromagnetic structure, used to accelerate or deflect the beam.
- **"Bunch":** a cluster of particles which is stable with respect to the accelerating RF
- **"Dipole":** magnet with a uniform magnetic field, used to bend particles
- **"Quadrupole":** magnet with a field that is ~linear near the center, used to focus particles
- **"Lattice":** the magnetic configuration of a ring or beam line
- **"Beta function (β)":** a function of the beam lattice used to characterize the beam size.
- **"Emittance (ε)":** a measure of the spatial and angular spread of the beam

$$\text{size of beam} \propto \sqrt{\varepsilon\beta_T}$$

- **"Tune":** number of times the beam "wiggles" when it goes around a ring. Fractional part related to beam stability.
- **"Longitudinal Emittance":** area of the beam in the Δt-ΔE plane. Constant with energy and adiabatic RF voltage change
- **"Luminosity":** rate at which particles "hit each other". Constant of proportionality between cross-section and rate.

Some further reading

- The definitive book on basic accelerator physics is
  - Syphers and Edwards, "An Introduction to the Physics of High Energy Accelerators"
- Other good books are:
  - S.Y. Lee, "Accelerator Physics"
  - Helmut Weideman, "Particle Accelerator Physics"
- Some good web resources:
  - Bill Barletta's notes from the undergraduate USPAS course
    - http://uspas.fnal.gov/materials/09UNM/UNMFund.html
  - Gerry Dugan's notes from the graduate USPAS course
    - http://www.lns.cornell.edu/~dugan/USPAS/
- Of course, you could always take the USPAS course
  - http://uspas.fnal.gov/